Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

Course: MPZ 3231-Engineering Mathematics IA
Academic Year – 2012/2013 **Assignment No.01**

Instructions

- Answer all questions
- Write your address back of your answer scripts
- Use both sides of paper when you are doing assignment.
- Please send the answer scripts of your assignment on or before the due date to the following address.

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[redacted]

(1) (a) Suppose ABCDEF is the vertices of a regular hexagon. Let $\overline{AB} = \underline{a}$ and $\overline{BC} = \underline{b}$.

(i) Express the vectors $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$ in terms of $\underline{a}$ and $\underline{b}$.

(ii) Find the resultant of the forces represented by the vectors in (i), in terms of $\overline{AD}$.

(b) If O is any point within triangle ABC and P, Q and R are midpoints of the sides AB, BC, and CA respectively. Prove that $\overline{OA} + \overline{OB} + \overline{OC} = \overline{OP} + \overline{OQ} + \overline{OR}$

(c) $ABCD$ is a parallelogram with P and Q the midpoints of sides BC and CD, respectively. Prove that AP and AQ trisect diagonal BD at points E and F.

(d) In the parallelogram $ABCD$, show that

(i) $\overline{AC} + \overline{BD} = 2\,\overline{BC}$ and

(ii) $\overline{AC} - \overline{BD} = 2\,\overline{AB}$

(2) (a) Define the scalar product of two vectors.

(b) If $\underline{a}, \underline{b}, \underline{c}$ be three vectors such that $\underline{a} + \underline{b} + \underline{c} = 0$, $|\underline{a}| = 3$, $|\underline{b}| = 4$, $|\underline{c}| = 5$, show that $\underline{a}.\underline{b} + \underline{b}.\underline{c} + \underline{c}.\underline{a} = -25$.

(c) If G be the centroid of a triangle ABC, show that

$$AB^2 + BC^2 + CA^2 = 3\,(AG^2 + BG^2 + CG^2)$$

(d) If D be the mid-point of the side BC of a triangle ABC, prove that

$$AB^2 + AC^2 = 2\,(AD^2 + BD^2)$$

(3) (a) Define the vector product of two vectors.

(b) If $|\underline{a}| = 3$ and $|\underline{b}| = 4$, find the values of μ for which the vectors $\underline{a} + \mu\underline{b}$ and $\underline{a} - \mu\underline{b}$ will be perpendicular to each other.

(c) Determine the values of λ and μ so that the following vectors are collinear.

$-3\,\underline{i} + 4\,\underline{j} + \mu\underline{k}$ and $\lambda\,\underline{i} + 8\,\underline{j} + 6\underline{k}$

(d) Determine the value of the constant d, so that the following vectors are coplanar.

$2\,\underline{i} - \underline{j} + \underline{k}$, $\underline{i} + \underline{j} + d\underline{k}$ and $3\,\underline{i} - 4\,\underline{j} + \underline{k}$

(e) Find a vector $\underline{d}$ which is perpendicular to both following vectors

$\underline{a} = 4\,\underline{i} + 5\,\underline{j} - \underline{k}$ and $\underline{b} = \underline{i} - 4\,\underline{j} + 5\underline{k}$ and which satisfies the relation $\underline{d}.\underline{c} = 21$ where $\underline{c} = 3\,\underline{i} + \underline{j} - \underline{k}$.

(4) (a) Given the matrices $B = \begin{bmatrix} 2 & 4 \\ 7 & 8 \end{bmatrix}$ and $(AB)^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$, Find A^{-1}.

(b) Write $A = \begin{bmatrix} 4 & 5 \\ 1 & 3 \end{bmatrix}$ as the sum of a symmetric matrix B and a skew-symmetric matrix C.

(c) Find real numbers x, y, z such that A is Hermition, where

$$A = \begin{bmatrix} 3 & x + 2i & yi \\ 3 - 2i & 0 & 1 + zi \\ yi & 1 - xi & -1 \end{bmatrix}$$

(d) Compute the product AB for the given matrices. Whenever possible, compute the product BA. $A = \begin{bmatrix} 2 & 0 & 5 \\ 1 & -2 & 4 \\ -3 & 1 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 8 & 1 \\ 5 & -1 \end{bmatrix}$

(e) Using Cramer's Rule, solve the following system of linear equations.

$$2x - 5y + 2z = 2$$
$$x + 2y - 4z = 5$$
$$3x - 4y - 6z = 1$$

(5) (a) If $A = \begin{bmatrix} -x & 1 & 0 & 0 \\ 0 & -x & 1 & 0 \\ 0 & 0 & -x & 1 \\ -C_0 & -C_1 & -C_2 & -C_3 - x \end{bmatrix}$, find the det (A) using the definition.

(b) If $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1+c \end{bmatrix}$ and $B = \begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{bmatrix}$, find the det (A) and det (B) using the properties of determinants.

(c) Prove the following identities without evaluating the determinant.

(i) $\begin{vmatrix} 0 & a & b & c \\ a & 0 & c & b \\ b & c & 0 & a \\ c & b & a & 0 \end{vmatrix} \quad \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & c^2 & b^2 \\ 1 & c^2 & 0 & a^2 \\ 1 & b^2 & a^2 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(iii) $\begin{vmatrix} bcd & cda & dab & abc \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \\ a^4 & b^4 & c^4 & d^4 \end{vmatrix}$

(d) Find all t such that

(i) $\begin{vmatrix} t-4 & 3 \\ 2 & t-9 \end{vmatrix} = 0$ (ii) $\begin{vmatrix} t-1 & 4 \\ 3 & t-2 \end{vmatrix} = 0$